# PLANE WAVES IN NONHOMOGENEOUS PLASTIC MEDIA AND THEIR INTERACTION WITH OBSTACLES

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Media in which the propagation of strong disturbances is usually studied are in many cases nonhomogeneous.

The nonhomogeneity may show up in the presence of more or less clearly defined layers with different characteristics or in continuous variation of the properties from particle to particle; for example, with increasing distance from the free surface the moisture content and the density of soils usually increase monotonically. In this case a wave traveling from the free surface encounters along its path particles with different compressibility laws. With increase of the distance the compressibility of the medium ahead of the wave front decreases. Experiments show that for sufficiently intense reduction of the soil compressibility an increase of the pressure in a plane wave is observed in certain segments with increasing distance from the disturbance source (for example, the location of an explosive charge blast). For motion of the blast wave in the opposite direction we observe a pressure decrease which is faster than in a homogeneous medium.

Plane wave propagation in media with continuously varying properties has been studied in [1–5] and elsewhere. In the following we use the method of [5, 6] to solve the problem of plane one-dimensional shock wave (strong disturbance) propagation in a nonhomogeneous plastic medium whose properties vary continuously from particle to particle and also the problem of reflection of this wave from an obstacle. The assumption adopted that the volume of the medium is constant during unloading and reloading and the approximation of the uniaxial compression stress-strain $\sigma(\varepsilon)$ diagram (or the pressure-volume relation p(V)) by a piecewise linear function make it possible to obtain the solution in simple analytic form. It is shown that there can be a marked pressure increase as a plane wave propagates in nonhomogeneous media.

1. Wave propagation. We examine a medium whose p(V) compression diagram in some pressure range in each particle can be approximated by two straight-line elements (Fig. 1). The slope of the first element and the maximum value $p_s$ of the pressure corresponding to this element are the same for all the particles. The slope of the second element changes monotonically with increase of the space coordinate h

$$
\begin{aligned}
p &= -A_0^2[V_s(h) - V_0(h)], \qquad & p \leqslant p_s \\
p - p_s &= -A^2(h)\ [V(h) - V_s(h)], \qquad & p \geqslant p_s \\
p &= -\sigma, \qquad \varepsilon = \frac{V - V_0}{V_0} = \frac{\rho_0 - \rho}{\rho} & \qquad (1.1)
\end{aligned}
$$

The subscript 0 corresponds to the initial state of the medium, $p_0$ is assumed to be zero, $\sigma$ is the stress component in the wave motion direction, and $\rho$ is the density of the medium.

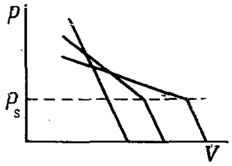


Fig. 1

We assume unloading and reloading to take place at constant volume:

$$\frac{\partial \varepsilon}{\partial t} = 0 \quad \text{or} \quad \frac{\partial V}{\partial t} = 0 \qquad (1.2)$$

We shall use the Lagrangian mass h and time t coordinates. The basic equations of motion in these coordinates have the form

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial h} = 0, \qquad \frac{\partial u}{\partial h} - \frac{\partial V}{\partial t} = 0, \qquad h = \int\limits_{x(0,t)}^{x(h,t)} \rho_0(x)\, dx \qquad (1.3)$$

Here u(h, t) is particle velocity and x is distance in length units.

Assume that at the initial section h = 0 of the medium at t = 0 the pressure increases stepwise to $p_m$ and then decreases as

$$p = f(t) \tag{1.4}$$

At t = 0 two shock fronts (Fig. 2) start from the initial section of the medium. The equation of the first wave front line is $h = A_0 t$.

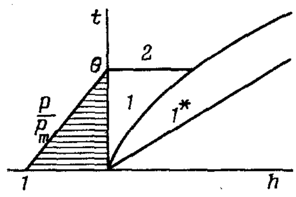

Fig. 2

The flow behind this wave front (region 1* in Fig. 2) is defined by the equations $p = p_s$, $A_0 u = p_s$.

Unloading of the medium takes place behind the second wave front (region 1 in Fig. 2). If (1.2) is satisfied the solution of (1.3) in the unloading region is known [6]:

$$u = \varphi(t), \qquad p = -h\varphi^{\cdot}(t) + \psi(t) \tag{1.5}$$

The particle velocity is independent of the space coordinate.

The problem reduces to determining from the boundary conditions the functions $\varphi(t)$ and $\psi(t)$ and also the front line $h_1(t)$. In the following we write these functions and also p(t) with subscript corresponding to the region number in the ht-plane.

The first boundary condition is that $p = f(t)$ for h = 0. Hence we find

$$\psi_1(t) = f(t) \tag{1.6}$$

The shock relations are satisfied at the front, which in h, t coordinates have the form

$$p - p_s = h_1^{\cdot 2}[V_s(h) - V(h)], \qquad u - u_s = h_1^{\cdot}(t)[V_s(h) - V(h)] \tag{1.7}$$

Comparing (1.7) to (1.1) we find the equation of the front line:

$$h^{\cdot}_1(t) = A(h), \qquad \int \frac{dh}{A(h)} = t + C, \qquad h_1(0) = 0 \tag{1.8}$$

If the relation A(h) is linear,

$$A(h) = A_1 + \varkappa h \tag{1.9}$$

then we obtain from (1.8) the equation of the front line $\varkappa h_1(t) = A_1(e^{\varkappa t} - 1)$.

For $\varkappa > 0$ the front velocity increases with increase of h; for $\varkappa = 0$ it remains unchanged. Figure 2 corresponds to the former case.

Let us find $\varphi_1(t)$. On the front by virtue of (1.5) and (1.7)

$$\begin{gathered} p - p_s = A(h)\ (u - u_s) \\ -h_1(t)\varphi_1^{\cdot}(t) + f(t) = h_1(t)\ [\varphi_1(t) - u_s] + p_s \end{gathered} \tag{1.10}$$

Integrating this equation, we find $\varphi_1(t)$ for the initial condition

$$\varphi_1(0) = \frac{f(0) - p_s}{A_1} + u_s \tag{1.11}$$

If for h = 0 the pressure is given by the linear function

$$p = p_m\left(1 - \frac{t}{\theta}\right) \tag{1.12}$$

and A(h) is given in the form (1.9), then the solution in region 1 has the form

$$\begin{aligned} u = \varphi_1(t) &= \frac{\varkappa}{A_1(1-e^{\varkappa t})}\left[(p_s - p_m)t + \frac{p_m t^2}{2\theta}\right] + \frac{p_s}{A_0} \\ p = -h\Big\{&\frac{\varkappa}{A_1(1-e^{\varkappa t})}\left[p_s - p_m\left(1-\frac{t}{\theta}\right)\right] + \frac{\varkappa^2 e^{\varkappa t}}{A_1(1-e^{\varkappa t})^2}\left[(p_s - p_m)t + \frac{p_m t^2}{2\theta}\right]\Big\} \\ &+ p_m\left(1 - \frac{t}{\theta}\right) \end{aligned} \tag{1.13}$$

If the medium is homogeneous, i.e., $\varkappa = 0$, then by passage to the limit we obtain the expressions found previously [6] for the particle velocity and pressure in region 1.

At the front $h_1(t)$ the particle velocity and pressure are defined in accordance with (1.13) by the equations

$$\begin{aligned} u(h) &= -\frac{1}{\varkappa h}\left\{(p_s - p_m)\ln\left(1 + \frac{\varkappa h}{A_1}\right) + \frac{p_m}{2\varkappa\theta}\left[\ln\left(1+\frac{\varkappa h}{A_1}\right)\right]^2\right\} \\ p(h) &= p_s + \left(1 + \frac{A_1}{\varkappa h}\right)\ln\left(1+\frac{\varkappa h}{A_1}\right)\left[p_m - p_s - \frac{p_m}{2\varkappa\theta}\ln\left(1 + \frac{\varkappa h}{A_1}\right)\right] \end{aligned} \tag{1.14}$$

At time $t = \theta$, when the pressure at the initial section drops to zero, region 2 develops (Fig. 2), in which further unloading of the medium takes place and the flow is defined by (1.5). Let us assume that at $t = \theta$ the second wave front has not caught up with the first wave front. Let us find the solution in the second region.

From the condition that p = 0 for h = 0 we find

$$\psi_2(t) = 0, \qquad u = \varphi_2(t), \quad p = -h\varphi_2^{\cdot}(t) \tag{1.15}$$

From the conditions at the boundary 2-1*, which corresponds to the shock front, it follows that the front line $h_2(t)$ is defined, as in region 1, by (1.8).

From the conditions at the boundary 2-1* we obtain

$$-h_2(t)\varphi_2^{\cdot}(t) - p_s = h_2^{\cdot}(t)\,[\varphi_2(t) - u_s] \tag{1.16}$$

After integrating this equation we find $\varphi_2(t)$. We find the constant of integration from the condition that the particle velocity be continuous at the boundary 2-1, i.e., from the condition $\varphi_2(\theta) = \varphi_1(\theta)$.

If the pressure at the initial section is given in the form (1.12) and A(h) is given in the form (1.9), after integrating (1.16) we obtain the solution in region 2 in the form

$$\begin{aligned} u = \varphi_2(t) &= u_s + \frac{\varkappa}{A_1(1-e^{\varkappa t})}\left(p_s t - \frac{p_m\theta}{2}\right) \\ p &= -\frac{\varkappa h}{A_1}(1-e^{\varkappa t})^{-2}\left[p_s(1-e^{\varkappa t}) + \varkappa\left(p_s t - \frac{p_m\theta}{2}\right)e^{\varkappa t}\right] \end{aligned} \tag{1.17}$$

It follows from (1.17) that at the wave front

$$\begin{aligned} u &= u_s - \frac{1}{h}\left[\frac{p_s}{\varkappa}\ln\left(1+\frac{\varkappa h}{A_1}\right) - \frac{p_m\theta}{2}\right] \\ p &= p_s + \left[\frac{p_m\theta}{2} - \frac{p_s}{\varkappa}\ln\left(1 + \frac{\varkappa h}{A_1}\right)\right]\frac{\varkappa h + A_1}{h} \end{aligned} \tag{1.18}$$

and the solution is completed.

2. **Wave interaction with an obstacle.** In certain media (soils, for example), the pressure $p_s$ usually does not exceed a fraction of the atmospheric pressure. We see from (1.13) that the presence of a weak wave with pressure

$p_s \ll p_m$ has no significant effect on the parameters of the strong disturbance traveling behind the wave. At the same time, in the case of interaction with an obstacle, account for the weak wave leads to the formation of a large number of small regions in which the definition of the flow is time consuming. Therefore, we solve this part of the problem using the approximation of the compression diagram by a single element (i.e., we assume $p_s = 0$).

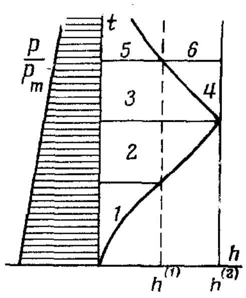


Fig. 3

The solution in region 1 (Fig. 3) was obtained above. Assume that further change of the acoustic resistance terminates at the section $h^{(1)}$, corresponding to this region, i.e., $A = A^*$ for $h \geqslant h^{(1)}$. Then region 2 develops. Let us find the solution in this region. The shock front line, which we denote by $h_2(t)$, is straight:

$$h_2(t) - h^{(1)} = A^* (t - t^{(1)}) \tag{2.1}$$

If the acoustic resistance changes in region 1 in accordance with (1.9), then

$$A^* = A_1 + \varkappa h^{(1)} \tag{2.2}$$

Unloading of the medium takes place in region 2 and we seek the solution in the form (1.5). From the condition at the section $h = 0$ we find that $\psi_2(t) = f(t)$. The conditions at the shock front are: $p = A^{*2}(V_0 - V)$, $u = A^*(V_0 - V)$. Hence we find

$$h_2^{\cdot}\varphi_2 = -h_2\varphi_2^{\cdot} + f(t), \qquad \varphi_2 = \frac{1}{h_2}\int f(t)\,dt, \qquad \varphi_2(t^{(1)}) = \varphi_1(t^{(1)}) \tag{2.3}$$

Let us examine the case in which the pressure at the initial section follows (1.12) and the acoustic resistance follows (1.9). Then we find from (2.3) the particle velocity and pressure in the form

$$\begin{gathered} u = \varphi_2 = \frac{1}{h_2(t)}\, p_m t\left(1 - \frac{t}{2\theta}\right) \\ \varphi_2^{\cdot} = \frac{1}{h_2^2}\left[h_2 p_m\left(1 - \frac{t}{\theta}\right) - h_2^{\cdot} p_m t\left(1 - \frac{t}{2\theta}\right)\right] \\ p = -h_2\varphi_2^{\cdot} + p_m\left(1 - \frac{t}{\theta}\right) \end{gathered} \tag{2.4}$$

The pressure at the wave front is

$$p = p_m \frac{A_1 + \varkappa h^{(1)}}{2h_2\theta}\left(\frac{h_2 - h^{(1)}}{A_1 + \varkappa h^{(1)}} + t^{(1)}\right)\left(2\theta - t^{(1)} - \frac{h_2 - h^{(1)}}{A_1 + \varkappa h^{(1)}}\right) \tag{2.5}$$

Let a stationary obstacle be located at the section $h^{(2)}$.

The two regions 3 and 4 (Fig. 3) are formed upon wave reflection from the obstacle. Unloading and loading of the medium take place in region 3. For $h > h^{(1)}$ loading takes place up to a value of the pressure which is different for the different particles but is equal to the pressure reached at the incident wave front $h_2(t)$. In this case $\partial V/\partial t = 0$ and the solution has the form (1.5).

It follows from the condition at section $h = 0$ that $\psi_3(t) = f(t)$. At the boundary $h_3(t)$ with region 4 we have in accordance with (2.5)

$$\begin{gathered} p_3(h_3) = -h_3(t)\varphi_3(t) + f(t) = \frac{p_m}{2h_3(t)\theta}\left(h_3(t) - h^{(1)} + A^* t^{(1)}\right) \\ \times\left(2\theta - t^{(1)} - \frac{h_3(t) - h^{(1)}}{A^*}\right) \end{gathered} \tag{2.6}$$

Unloading of the medium takes place in region 4 behind the pressure jump at the boundary $h_3(t)$. It follows from the condition that the obstacle be stationary that in region 4 the particle velocity equals zero and the pressure depends only on the time,

$$\varphi_4 = 0, \qquad p_4 = \psi_4(t) \tag{2.7}$$

Thus, to define the flow in regions 3 and 4 we must find $\varphi_3(t)$, $h_3(t)$, $\psi_4(t)$.

The conditions at the boundary $h_3(t)$ yield

$$p_4 - p_3 = h_3^{\cdot}(t)\varphi_3(t), \qquad -\varphi_3(t) = h_3^{\cdot}(t)(V_3 - V_4) \tag{2.8}$$

Assuming that $p_3 = -A^{*2}(V_3 - V_4)$, $p_4 = A^{*2}(V_4 - V_0)$, we then find the front line

$$h_3(t) = h^{(2)} - A^*(t - t^{(2)}), \qquad t^{(2)} = t^{(1)} + \frac{h^{(2)} - h^{(1)}}{A^*} \tag{2.9}$$

It follows from (2.8), (2.6), (2.7) that

$$\psi_4 + h_3\dot{\varphi}_3 - f = \dot{h}_3\varphi_3, \qquad \psi_4 = p_3 + A^*\varphi_3 \tag{2.10}$$

$$\psi_4^{\cdot} = p_3^{\cdot}(h_3) + A^*\varphi_3^{\cdot}, \qquad p_3^{\cdot} = -\frac{dp_3}{dh_3}A^*$$

$$\psi_4^{\cdot} = A^*\left(-\frac{p_3}{h_3} - \frac{dp_3}{dh_3} + \frac{f}{h_3}\right) = \frac{p_m}{\theta}\left(1 - \frac{h^{(1)} - A^*(t - t^{(1)})}{h_3}\right)$$

From (2.10) and (2.9) we obtain

$$\psi_4^{\cdot} = \frac{2p_m}{\theta}\left(1 - \frac{h^{(2)}}{2h^{(2)} - h^{(1)} - A^*(t - t^{(1)})}\right) \tag{2.11}$$

Integrating and finding the arbitrary constant from the condition

$$\psi_4(h^{(2)}) = p_3(h^{(2)}) + A^*\varphi_2(t^{(2)}) \tag{2.12}$$

we determine the pressure in region 4:

$$p_4 = \psi_4 = \frac{2p_m}{\theta}\left\{t - t^{(2)} + \frac{h^{(2)}}{A^*}\ln\left[2 - \frac{h^{(1)}}{h^{(2)}}(t - t^{(1)})\right]\right\}$$

$$+ p_3(h^{(2)}) + A^*\varphi_2(t^{(2)}), \qquad \varphi_2(t^{(2)}) = p_m\frac{2\theta t^{(2)} - (t^{(2)})^2}{2\theta h^{(2)}} \tag{2.13}$$

$$p_3(h^{(2)}) = p_m\left[\left(1 - \frac{h^{(1)} + A^*t^{(1)}}{h^{(2)}}\right)\left(1 - \frac{h^{(2)} - h^{(1)} - 2A^*t^{(1)}}{2A^*\theta}\right)\right. \tag{2.14}$$

We find the particle velocity in region 3 from (2.6):

$$u = \varphi_3(t) = \int\frac{f(t) - p_3(h_3)}{h_3(t)}dt + C \tag{2.15}$$

We find the arbitrary constant from the condition that the velocity be continuous for $t = t^{(3)}$.

In accordance with (2.6) and (2.9) we substitute the expressions for $p(h_3)$ and $h_3(t)$ into (2.15). We replace integration with respect to t by integration with respect to $h_3$:

$$dt = -\frac{dh_3}{A^*}, \qquad t = t^{(2)} + \frac{h^{(2)} - h^{(3)}}{A^*} = t^{(1)} + \frac{2h^{(2)} - h^{(1)} - h_3}{A^*}$$

Then the particle velocity in region 3 is

$$\varphi_3 = -\frac{p_m}{A^*}\left\{\frac{3(h_3 - h^{(2)})}{2A^*\theta} - \frac{2h^{(2)}}{A^*\theta}\ln\frac{h_3}{h^{(2)}}\right.$$

$$\left.-\left[h^{(1)}\left(1 - \frac{t}{\theta}\right) + \frac{h^{(1)2}}{2A^*\theta} - A^*t^{(1)}\left(1 - \frac{t^{(1)}}{2\theta}\right)\right]\left(\frac{1}{h_3} - \frac{1}{h^{(2)}}\right)\right\} + \varphi_2(t^{(2)}) \tag{2.16}$$

Thus the flow in regions 2, 3, and 4 has been determined.

At the time $t^{(3)}$ the wave front reflected from the obstacle reaches the section $h^{(1)}$. Then the new regions 5 and 6 appear.

Unloading of the medium and reloading to the value reached at the incident wave front take place in region 5. The difference from region 3 is that the initial loading laws are different for all the particles. As a result of this the boundary 5-6 is curved. We denote it by $h_4(t)$. We find from the condition at section h = 0 and (1.5) that

$$\psi_5 = f(t), \qquad \varphi_5^{\cdot} = \frac{f(t) - p_5(h_4)}{h_4}, \qquad p_5(h_4) = p_4(h_4) \tag{2.17}$$

The front line $h_4(t)$ is found from the conditions

$$p_6 - p_5 = h_4^{\cdot}\varphi_5, \quad \varphi_6 = 0, \quad -\varphi_5 = h_4^{\cdot}(V_5 - V_6) = \frac{h_4^{\cdot}(p_6 - p_5)}{(A_1 + \varkappa h_4)^2} \tag{2.18}$$

Hence

$$h_4^{\cdot} = -(A_1 + \varkappa h_4), \qquad h_4(t) = \frac{A_1}{\varkappa}\left[\left(1 + \frac{\varkappa h}{A_1}\right)e^{-\varkappa(t - t^{(3)})} - 1\right]$$
$$t^{(3)} = t^{(1)} + 2t^{(2)} = t^{(1)} + 2\frac{h^{(2)} - h^{(1)}}{A_1 + \varkappa h^{(1)}} \tag{2.19}$$

We substitute t determined from (2.19) into (2.17), and we replace integration with respect to t by integration with respect to $h_4$:

$$dt = -\frac{dh_4}{A_1 + \varkappa h_4}, \qquad t = t^{(3)} - \frac{1}{\varkappa}\ln\frac{A_1 + \varkappa h_4}{A_1 + \varkappa h^{(1)}}$$

Then

$$\varphi_5(t) = -p_m\int\left\{\frac{1}{(A_1 + \varkappa h_4)h_4}\left(1 - \frac{t^{(3)}}{\theta} + \frac{1}{\varkappa}\ln\frac{A_1 + \varkappa h_4}{A_1 + \varkappa h^*}\right)\right.$$
$$\left. - \frac{1}{\varkappa h_4^2}\left[\ln\frac{A_1 + \varkappa h_4}{A_1} - \ln\frac{A_1 + \varkappa h_4}{A_1}\right]^2\right\}dh_4 + C$$

The constant is found from the condition that the particle velocity be continuous at the boundary of regions 3 and 5.

In region 6 the particle velocity is zero and the pressure is found from (2.18) and (2.19):

$$p_6 = p_5(h_4) - h_4^{\cdot}\varphi_5 = p_5(h_4) + (A_1 + \varkappa h_4)\varphi_5$$

Thus the solution in regions 5 and 6 has been found. However, the integral defining the function $\varphi_5(t)$ cannot be expressed in elementary functions, and therefore the determination of the flow in these regions must be carried out numerically.

3. Discussion of computational results. The physical essence of the process of wave propagation and interaction in nonhomogeneous media can be explained on the basis of the expressions obtained above. Let us examine the case in which the pressure at the initial section of the soil corresponds to (1.12) and the acoustic resistance variation corresponds to (1.9). Let us first evaluate the possible values of $\varkappa$. For example, we assume that in the layer adjacent to the initial section $\rho = 1.6 \cdot 10^3$ kg/m$^3$, and the plastic wave velocity C = 80 m/sec. At the distance 20 m we have $\rho = 2 \cdot 10^3$ kg/cm$^3$ and c = 420 m/sec. Taking a linear variation of density with distance, we find that $\varkappa = 20$ sec$^{-1}$. The increase of c and $\rho$ in individual segments in other dense media may take place more intensely.

In Fig. 4 curves 1, 2, and 3 correspond to the maximum pressure at the incident and reflected wave fronts. It is assumed that the properties of the medium do not change with depth beginning at the section $h^{(1)}$. Then $h^{(1)}/A_1 = 0.0858$ sec. At the section $h^{(2)}$ there is a stationary obstacle made from incompressible material $h^{(2)}/A_1 = 0.154$ sec.

Curve 1 corresponds to the pressure for $\theta = 0.1$ sec, $\varkappa = 20$ sec$^{-1}$, curve 2 is for $\theta = 1$ sec, $\varkappa = 0$ (homogeneous medium), and curve 3 is for $\theta = 1$ sec, $\varkappa = 20$ sec$^{-1}$. In the nonhomogeneous media $t^{1} = 0.05$ sec, $t^{(2)} = 0.075$ sec, and in the homogeneous medium $t^{(1)} = 0.0858$ sec, $t^{(1)} = 0.154$ sec.

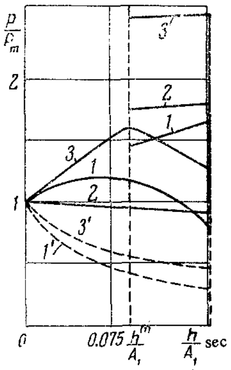


Fig. 4

We see from the curves that with increasing distance from the initial section for $\varkappa > 0$ increase of the pressure may take place. With reduction of $\theta$ the magnitude of the increase diminishes. Upon transition into the region $\varkappa = 0$ the pressure at the front begins to decrease with distance. With approach to the obstacle the pressure for $\theta = 1$ sec still has not decreased to $p_m$, while for $\theta = 0.1$ sec it decreases to 0.82 $p_m$.

Upon reflection from the obstacle the pressure increases stepwise by a factor of two. The pressure at the reflected wave front decreases with approach to the initial section. For a smaller value of $\theta$ the decrease is more intense.

Curve 1' corresponds to the particle velocity for $\theta = 0.1$ sec, $\varkappa = 20$ sec$^{-1}$, and curve 3' is for $\theta = 1$ sec, $\varkappa = 20$ sec$^{-1}$. In the homogeneous medium the curves of $p/p_m$ and $A_1 u / p_m$ coincide.

In all cases the particle velocity decreases with distance. In the nonhomogeneous medium the particle velocity decreases more rapidly with distance than in a homogeneous medium. With increase of $\theta$ the rate of velocity decrease diminishes. The particle velocity in the reflected wave equals zero.

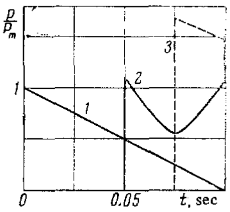


Fig. 5

Figures 5, 6, and 7 show curves of the relation $p(t)/p_m$ for certain sections of the medium. These figures correspond, respectively, to the cases $\theta = 0.1$ sec, $\varkappa = 20$ sec$^{-1}$; $\theta = 1$ sec, $\varkappa = 0$: $\theta = 1$ sec, $\varkappa = 20$ sec$^{-1}$. In all the figures curve 1 is the pressure for $h = 0$, curve 2 is for $h = h^{(1)}$, and curve 3 is for $h = h^{(2)}$.

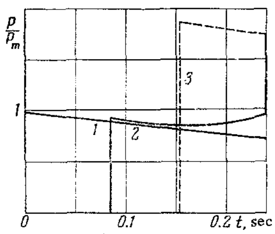


Fig. 6

At all sections behind the incident wave front there is a decrease of the pressure, accompanied by unloading of the medium. This decrease also continues after $t = t^{(2)}$. However, the zone of pressure decrease entering region 3 (i. e., for $t > t^{(2)}$), at all points other than those lying close to the section h = 0, is rapidly replaced by a zone of pressure increase, which takes place with $\partial V/\partial t = 0$. Behind the pressure jump the pressure on the reflected wave front $h_3(t)$ again decreases.

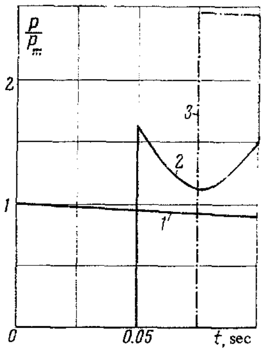


Fig. 7

Thus, in nonhomogeneous media with $\varkappa > 0$ there is an increase of the pressure as the wave propagates. With increase of $\varkappa$ the intensity of the pressure rise increases. The maximum wave load acting on an obstacle may exceed significantly the load acting in a homogeneous medium for the same pressure variation law on the free surface. The particle velocity is less in nonhomogeneous media than in homogeneous media.

The solution was carried out without account for the viscous properties of the medium, i. e., the dependence of the $\sigma(\varepsilon)$ diagram on the deformation rate. Account for this factor leads to increase of the energy losses in the wave and consequently to attenuation of the wave with distance [5].

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